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10CS34

Third Semester B.E. Degree Examination, June/July 2013
Discrete Mathematical Structure

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Define a set, proper subset and power set, with an example for each. **(06 Marks)**

b. A survey of 500 television viewers of a sports channel produced the following information : 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not any of the three kinds of games.
   i) How many viewers in the survey watch all three games?
   ii) How many watch exactly one of the sports? **(07 Marks)**

c. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5. What is the probability that he will get both the contracts? **(07 Marks)**

2 a. Define a tautology and contradiction. Prove that the proposition $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow (p \vee q) \rightarrow r]$ is a tautology. **(06 Marks)**

b. Prove the following logical equivalences using laws of logic
   i) $[(\sim p \vee \sim q\} \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$
   ii) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$. **(07 Marks)**

c. Define converse, inverse and countrapositive of a conditional with truth table. Also state the converse, inverse and contrapositive of the following statement.
   "If a triangle is not isosceles, then it is not equilateral". **(07 Marks)**

3 a. Write down the following proposition in symbolic form and find its negation :
   "All integers are rational numbers and some rational numbers are not integers". **(06 Marks)**

b. Show that the following argument is valid. NO engineering student of first or second semester studies logic
   <u>Anil is an engineering student who studies logic</u>
   ∴ Anil is not in second semester. **(07 Marks)**

c. Give i) a direct proof ii) an indirect proof iii) proof by contradiction for the following statement :
   "If n is an odd integer, then n + g is an even integer". **(07 Marks)**

4 a. State the induction principle. Prove the following result by mathematical induction :
   "For every positive integer n, 5 divides $n^5 - n$". **(06 Marks)**

b. Find an explicit definition of the sequence defined by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$. **(07 Marks)**

c. If L_0, L_1, L_2 - - - - are Lucas numbers, prove that

$$L_n = \left[\frac{1+\sqrt{5}}{2}\right]^n + \left[\frac{1-\sqrt{5}}{2}\right]^n.$$

(07 Marks)

PART – B

5 a. For any non-empty sets A, B, C prove the following :

i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

ii) $A \times (B - C) = (A \times B) - (A \times B)$. **(06 Marks)**

b. Define a binary relation. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by $a^R b$ if and only if a is a multiple of b. Represent R as a set of ordered pairs. Draw the digraph of R. Write the matrix of R. **(07 Marks)**

c. Define an equivalence relation. Let N be the set of all natural numbers. On N × N, the relation R is defined as $(a, b)^R (c, d)$ if and only if a + d = b + c. show that R is an equivalence relation. Find the equivalence class of the element (2, 5). **(07 Marks)**

6 a. Let $f : z \to z$ be defined by f(a) = a + 1 for $a \in z$. Show that f is a bijection. **(06 Marks)**

b. Find the number of ways of distributing four distinct objects among three identical containers with some container (s) possibly empty. **(07 Marks)**

c. If $f : A \to B$, $g : B \to C$ and $h : C \to D$ are three functions then prove that (hog) of = ho(gof). **(07 Marks)**

7 a. Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2} ab$. Show that (G *) is an abelian group. **(06 Marks)**

b. State and prove Lagrange's theorem. **(07 Marks)**

c. The word C = 1010110 is sent through a binary symmetric channel. If p = 0.02is the probability of incorrect receipt of a signal, find the probability that C is received as r = 1011111. Determine the error pattern. **(07 Marks)**

8 a. The parity check matrix for an encoding function $E : z_2^3 \to z_2^6$ given by

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine the associated generator matrix

ii) Does this code correct all single errors in transmissions? **(06 Marks)**

b. For the following encoding function, find the minimum distance between the code words. Indicate the error – detecting and error –correcting capabilities of each code

$E : z_2^3 \to z_2^6$ defined by

E(0 0 0) = 000111	E(0 0 1) = 001001
E(0 1 0) = 010010	E(0 1 1) = 011100
E(1 0 0) = 100100	E(1 0 1) = 101010
E(1 1 0) = 110001	E(1 1 1) = 111000.

(07 Marks)

c. Prove that the set z with binary operations ⊕ and ⊙ defined by x ⊕ y = x + y – 1 and x ⊙ y = x + y – xy is a commutative ring. **(07 Marks)**

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